High Level Mathematical Thinking : Experiments with High School and Under Graduate Students using Various Approaches and Strategies

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ABSTRACT

This paper presents the findings from a three-year study (2003-2005) at Universitas Pendidikan Indonesia (UPI) by using various innovative approaches and strategies conducted to investigate students' abilities in high level mathematical thinking and dispositions. The study employs thirteen experimental control group designs involving a number of high school and under graduate students. The instruments of each study consist of a set of non-routine essay test, an attitude scales, and an observation sheet for students' activities during the lesson. The study found that : a) In general, students' high level mathematical thinking abilities were classified as mediocre. However, teachers stated that those abilities were important to be learned by students at all levels of education ; b) Ten out of thirteen sub-studies found that innovative approaches were able to improve students' high level mathematical thinking far better than those of conventional approaches. c) Students performed positive mathematical dispositions, where during the lessons they actively reinvented mathematical concepts and they had high interest in cooperative-work ; d) The innovative approaches tended to be more influential in improving students' high level mathematical thinking than other variables such as school categories and students' prior knowledge. e) Considering gender, two sub-studies reported inconsistent findings of students' mathematical thinking abilities. f) Teachers had willingness to implement the innovative approaches.

Key words : mathematical thinking, mathematical dispositions, problem-based learning, discursive approach, metacognitive approach, indirect-direct approach, deductive-inductive approach, e-learning, multimedia, think-talk-write strategy. transactional reading strategy.

A. Background

Mathematics education for elementary to tertiary education is aimed at fulfilling today and future demands of human life. The first objective directs mathematics teaching toward the understanding of mathematics

ideas, concepts, principles and rules for solving simple problems. The second objective is oriented more at developing high level and creative thinking and dispositions of mathematics.

Many studies of high level mathematical thinking have focused more on the view of mathematics as an active, dynamic, generative and explorative process than on that of mathematics as a deductive process (Schoenfeld, in Heningsen dan Stein, 1997). The first view of mathematics teaching is based on constructivism. In the beginning, students reinvent concepts inductively and then they prove the truth deductively. Those inductive and deductive processes take place cyclically. Viewed from the realistic mathematics education, the first process is called horizontal mathematization and the second is vertical mathematization (Treffers, in Heuvel-Panhuizen, 2000).

Some writers offered various approaches to encourage the development of high level and creative thinking in mathematics (Chamot, 1993, Cooney, Sanchez, and Ice, 2001, Peterson in Grouws, Cooney, and Jones, 1987, Thomas 1998, Williams, 2002). Their opinion was based on the view that knowing mathematics is doing mathematics. The view suggested that learning mathematics should be emphasized on doing math not on memorizing it. For example, while students are solving an open-ended problem, they need to be encouraged to investigate, explore, communicate, and justify the elements of the problem, and compile a conjecture.

Peterson (Grouws, Cooney, & Jones, 1987) adopted a learning model in a small group by presenting non-routine and metacognitve tasks. Chamot (1993) suggested the use of scaffolding, interactive and metacognitive approaches. Thomas (1998) created some activities such as discovery and creative learning, constructive criticism, and saw things from a different point of view. Like Thomas, Williams (2002) suggested presenting high level tasks that encourage students' creativity. Finally Cooney, Sanchez, and Ice (2001) recommended a reformation of learning mathematics from an imitated learning to a meaningful learning mode.

Some studies investigated students' high level mathematical thinking abilities by using various approaches (Hendriana, 2002 ; Kariadinata, 200 ; Sudrajat, 2000 ; Sugandi, 2001 ; Sumarmo, Suryadi, Rukmana, Dasari, and Suhendra, 1998, 1999, 2000 ; Wardani, 2002 ; Yaniawati, 2001). Sumarmo et. al. (1998, 1999, 2000) reported that using scaffolding in a small group learning, elementary school students were able to obtain their high level mathematical thinking and dispositions. Other studies using various innovative approaches found that there are improvement of senior high students' abilities such as problem posing and solving, analogy, communication, and connections. The students performed positive attitude toward learning mathematics as well (Hendriana, 2002 ; Kariadinata, 2001 ; Sudrajat, 2000 ; Sugandi, 2001 ; Wardani, 2002 ; Yaniawati, 2001).

The above findings were in line with the assumption that innovative teaching would give more opportunities to students to obtain high level mathematical thinking abilities and dispositions than the conventional teaching. The findings encourage researchers to conduct experimental studies by using various innovative teaching approaches to fit the mathematical thinking abilities for students at all levels of education.

B. Objectives of the study

The objectives of this study are :

(1) To develop teaching-learning materials and a series of instruments to fit certain kinds of mathematical thinking abilities and teaching-learning approaches for students of certain school levels.

(2) To analyze the quality of students' high level mathematical thinking abilities, students' mathematical dispositions and students' and teachers' opinions toward the innovative teaching approaches.
(3) To analyze students' activities during the lessons as well as the advantages and disadvantages of each teaching-learning approach.

C. Theoretical Review

1. High level mathematical thinking : characteristics and significance for students

There are various notions of mathematics where they depend on how one uses mathematics in one's activities. Although people use mathematics in different ways, they always use mathematics in their daily life. Viewed from Realistic Mathematics Education, the situation illustrates a characteristic of mathematics as human activities. Another characteristic is that mathematics is a deductive process, logical and axiomatic reasoning, which includes inductive processes such as compiling hypotheses, estimations, and conjectures. The inductive process is called horizontal mathematization and the deductive process is vertical mathematization (Treffers in Heuvel-Panhuizen, 2000). Based on the arrangements of their elements, mathematics is called a structured and systematic science. In addition to those characteristics, mathematics has also many specific features such as efficient symbols, beautiful orderliness, and quantitative analysis ability.

According to the depth and complexity of its activities, mathematical thinking is classified into two levels, low level and high level thinking. Doing simple arithmetic operations, applying rules directly, working on algorithm tasks are classified as low level thinking. On the other hand, meaningful understanding, conjecture compiling, analogy making and generalization, logical reasoning, problem solving, and mathematical communication and connection are classified as high level mathematical thinking (Webb and Coxford, 1993). Some writers used various terms for high level mathematical thinking. Champagne (1990) views it as high order cognitive skills, while Draper (1992) regards it as a structured, dynamic, generic, scicientific, closed-loop, and continum thinking. The other terms include metacognitive process (Davidson, Deuser, & Stemberg, 1994), and critical, creative, and constructive thinking (Thomas & Albee, 1998 ; Williams, 2002).

High level mathematical tasks are non-routine processes. Therefore, to solve the tasks, students should have strong motivation, enthusiasm, and willingness to solve them. Those affective features are called mathematical dispositions. Despite improving mathematical dispositions, regular exercises to high level mathematical tasks will stimulate other excellent attitudes to grow as well. The attitudes include being patient, careful, self-confident, tolerant, and aware of the beauty of language, symbols, and orderliness of mathematics. From this point of view, it is understandable that the development of high level mathematical thinking abilities and disposition are relevant to the attempt of achieving the objectives of mathematics learning for students at all levels of education.

2. Teaching approaches that fit the development of high level mathematical thinking and disposition of students

Building on Piaget's theory, Fischer (1980), Fischer and Bullock (1981), and Fischer and Pipp (1984) developed a similar theory. They stated that a child cognitive development is influenced by internal and external variables. So, it is possible to design a teaching-learning process to improve students' high level

mathematical thinking abilities. Tyler (1991) developed three key questions : (1) How can we help students to learn ? (2) What kinds of learning experiences should we prepare to encourage students to learn, and (3) How can we organize learning experiences so that they become significant cumulative influence ?

Piaget (Bell, 1978) stated that a child intellectual development is a series of assimilation and accomodation processes of information into the child's mental schemata. He further explained that the child's mental development is influenced by her or his maturity, physical experiences, logico-mathematical experiences, social transmission, and equilibrium. Fischer (1980) believed that the development of a child's intelectual abillity is formed through interaction and coordination of a new knowledge with its environment. Therefore to help students learn better we have to provide learning experiences to fit the five factors. Then, a question is raised : What kind of learning experiences should we provide to help students pose high level thinking abillity ?

In addition to the suggestions mentioned in Part A, there are other teaching-learning approaches that facilitate the development of high level mathematical thinking. They include discursive, metacognitive, problem- based, indirect-direct, inductive-deductive approaches, e-learning, and learning by using multimedia. Discursive mathematics teaching-learning involves mathematical communication activities such as writing mathematical essays, discussions, and mathematical debates in class situations. Therefore, the task of teachers is to create the class situation that supports mathematical discourses. This includes posing problems and questions, open-ended tasks, and non-routine tasks, inviting students to communicate actively and to think critically. A Metacognitive model offers a mathematical task that invites students to think deeply, to make connection among variables, to self-inquire, and to make a conclusion or judgment rationally.

A problem-based, an indirect-direct, and an inductive-deductive approach have similar characteristics i.e mathematical concepts are not presented in an end-form, instead the concepts have been constructed by the students. Teachers present a contextual problem solving task (in problem based approach), a contextual illustration that involves a mathematical idea (in indirect-direct approach), and some examples and non-examples of certain mathematical concepts or processes (in inductive-deductive approach).

E-learning and multimedia approaches have a similar characteristic to exploit ICT. E-learning exploits electronic services connected to internet, where students select learning materials to fit their needs. Some of the advantages of e-learning are : a) easier, wider, and borderless communication and discussions between teacher and students, b) accelerating innovasion process, c) flexible orderline of learning time, d) more efficient process of learning, and e) enabling students to review the learning materials any time and any place and to save them into their computers. Despite those advantages, there are some disadvantages of this e-learning approach such as : a) because students work individually, social values may be obstructed, b) teachers have to follow the development of technology in education continuously, c) students who are less motivated tend to fail, and d) internet connection is not avalilable in some schools. Some recomendations to succeed in e-learning are that students should : a) have a high self regulated learning, b) have their own objectives, c) select learning materials and ways of learning, d) select and solve learning tasks, e) reflect and self-evaluate their learning progress. Those conditions require teachers to develop various learning materials to fit the students' needs.

Some advantages of multimedia approach are that : a) it provides an interactive process, and facilities for feed back, b) it offers students' opportunities to select certain topics, c) it gives systematic control fasilities, d) it creates a dialog : students-teacher, and among students, and e) it plays a role as a teacher who has

capabilities to create some pictures, sounds, and events lively.

Despite the advantages and disadvantages of each teaching approach, in selecting a teaching approach we have to consider the changes of view of mathematics teaching and learning as well. Those changes are : a) from a classroom as a group of individual to a learning community, b) from similar services for all students to individual services to fit student's needs, strength and weakness, c) from teacher as an instructor to a motivator, facilitator, and manager of learning, d) from emphasizing on memorizing to emphasizing on understanding, reasoning, problem solving, connecting, and communicating, e) from teacher's individual to cooperative work, f) from mathematics as a collection of concepts to mathematics connecting many concepts, and g) from competitive situation to cooperative situation.

Other important suggestions in selecting a teaching approach include the followings. a) presenting relevant mathematics tasks, b) managing mathematical discourses, c) creating learning situations to develop mathematical power, and d) analyzing learning situations. Suggestions for evaluation are concerned with : a) the notion that evaluation of student learning is an integral part of teaching-learning process, b) the use of various methods of measurement, c) the development of various kinds of tests to fit the process and knowledge to be evaluated, and d) the match of the aspects that will be measured by the curriculum objectives.

D. Designs, learning materials, and instruments of the study

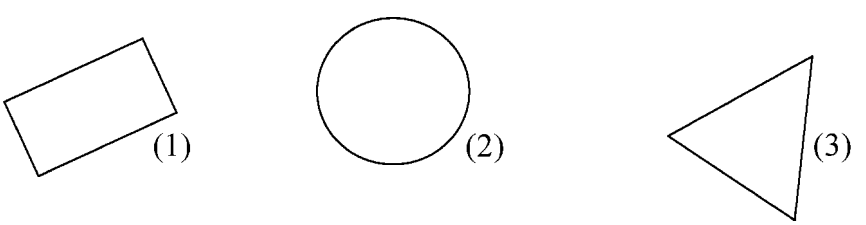
The study reported here was conducted at Universitas Pendidikan Indonesia (UPI) for three years (2003-2005) to investigate students' abilities in mathematical thinking and dispositions by using various approaches and strategies. The study consisted of fourteen sub-experimental control group designs involving a number of : high schools students, undergraduate students, and student-teachers. Each experiment adopted a certain innovative teaching approach, and administered a set of non-routine essay tests, a set of attitude scales, and a set of observation sheets for students' activities during the lesson. All learning materials and instruments were developed specifically to fit the objectives of each experiment.

In the following, we present examples of learning materials and item tests in some sub-studies.

Example 1. Learning material of problem-based approach for junior high school students (Herman, 2006)

The length of a fence

The government invites some enterprises to fence a piece of land. Each enterprise will fence the land by no more than 20 km in length. You are asked to fence the land so that you have the maximum area.

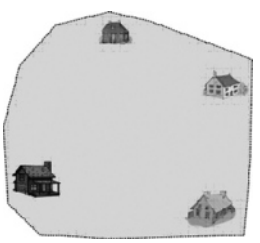


For example the land is a quadrangle (Figure 1)

A. Sketch some quadrangles with 20 km in length of the periphery and write the length and the width (in integers) of each of them
B. Write the length and the width of each quadrangle of point A in a table
C. Draw a graph that illustrates the relation between the length of one side and the area of the quadrangle. What is the form of the graph ?
D. If you want to have the maximum area of the land, what is the measure of the quadrangle ? Explain how you get it by using the graph you have made.

Answer the similar questions above (for Figure 2, and Figure 3), and make conclusions about them.

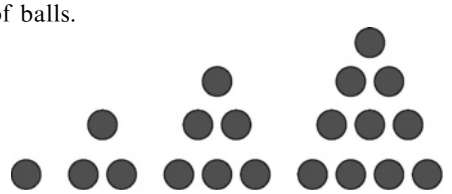
Example 2 : Learning material of indirect approach for junior high school students (Suryadi, 2005)



This picture is a sketch of housing.
The owners of the four houses want to have a garden surrounding their houses.
A bench will be placed in the garden so that it has the same distance to each house.
Locate the bench and explain the way you get it.

Example 3 : Learning material of problem-based learning for senior high school students (Ratnaningsih, 2003)

Observe the number of balls.



Pattern number :	1	2	3	4	…	n
Number of balls :	1	3	?	?		?

How many balls in pattern-3, and pattern-4 ? How do you get it ? Discuss with your friends ! Write down the number of balls in pattern-n. It is called general n-th term of pattern-n. This process is an example of generalization reasoning.
Give other examples. From those examples, write down the definition of generalization reasoning.

Examples 4 : Item test of critical thinking in mathematics for elementary student teacher (Mayadiana, 2004)

Mr. Adam is calculating the production cost of key-rings. Each ring consists of two different letters regardless of the order of the letters. Samples of Mr. Adam's data are included in the following table.

Number of letters	Number of the two letters
4	12
5	20
6	30
7	42
8	56

Based on those data, Mr. Adam estimates that from 50 letters he will get 2450 formation of two letters.
His estimation is : (select the true answer and give your eksplanation)

a. It is absolutely true
b. It may be true
c. It does not have enough data
d. It may be false
e. It is absolutely false

Explanation ·····

Example 5 : Item test of mathematical reasoning (analogy) for senior high school students (Permana, 2004)

From Jakarta to Bandung there are two routes of bus-line, and from Bandung to Semarang there are three routes of bus-line The relation between the number of route of bus-line from Jakarta to Semarang via Bandung and number 6 is

Similar ⇨ The relation between the number of pairs of trouser and shirt of three coloured trousers (white, blue and black) and two coloured shirts (yelow, and red) and number :

a. 2 b. 3 c. 5 d. 6 e. 8

Explain mathematics idea included in the relations above.

Example 6 : Item test of mathematical multiple representations for junior high school students (Mudzakir, 2005)

Let's go for a trip!

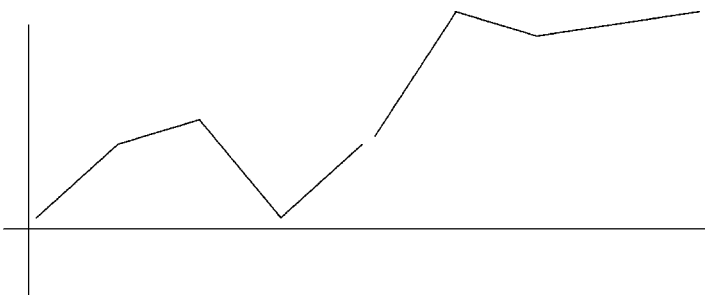
Car Rental : Laris
Rp 200.000,00 a day and Rp 3.000,00 per

Car Rental : Manis
Rp 150.000,00 a day and Rp 4.000,00 per

Mr. Tata and his family are going to rent a car for a holiday. Two Car-Rentals, Laris and Manis, offer their rates as above. Mr. Tata will choose the rate which gives him the greater advantages.
Write a mathematical model of each rate of the car-rental.
Suppose the distance of the trip is 45 km. Which car-rental will Mr. Tata choose ? Why ? Which one will he choose if the distance is 100 km ? Why ?

By using a graph, calculate the distance so that both rates are the same. What is the rate ?
Do the item (c) by using another way !

Example 7 : Item tests of mathematical communication for senior high school students (Muin, 2005)
Observe this diagram.



Write a title for the diagram, and complete it with relevant elements. Then compile an attractive story to fit the diagram.

E. Related studies

Henningsen and Stein (1997) propose that to develop students' capacity in doing math, the classroom situation has to be pleasant so that the students have a strong willingness to solve mathematical tasks. They suggest that teachers : a) select the tasks relevant to the students' interest, b) employ scaffolding encourage students to explain and compile a meaningful connection.

A number of studies using various strategies with high schools and under graduate students reported positive findings of students' mathematical thinking abilities (Afgani, 2004, Aminah, 2002, Ansyari, 2004, Hendriana, 2002, Kadir, 2000, Kariadinata, 2001, Nindiasari, 2004, Rohaeti, 2004, Sugandi 2001, Sudrajat, 2002, Wardani, 2002, and Yaniawati, 2001). Those studies found that as a result of innovative approaches students' abilities of high level mathematical thinking were classified as mediocre. Afgani (2004) and Hamzah (2003) experimenting with junior high school students found that problem-based learning and problem-posing were more effective to improve students abillities than those of conventional approaches. In addition, Afgani found that by using a problem-based learning in a small group, students' communication and reasoning abilities were higher than those of conventional group.

Other findings of senior high school students were reported by Aminah (2002), Ansyari (2004),. Hendriana (2002), Kariadinata (2001), Nindiasari, (2004), Rohaeti, (2004), Sudrajat (2002), Sugandi (2001) ; Wardani (2002), and Yaniawati (2001). Among the findings included : (a) by using the open-ended approach students achieved higher scores on mathematical understanding and connection than those of conventional groups. Sudrajat (2002) using Survey, Question, Review, Write (SQRT) found that, students gained higher scores in mathematical communication, and they had positive view about the teaching process. Similar findings were that by using "port-folio" assessment, students' achievement was classified as good (Aminah, 2002), and by using reciprocal teaching, probing and scaffolding students' understanding and problem solving

abilities are higher than those of conventional group (Hendriana, 2002). In addition, Rohaeti (2004), by using IMPROVE approach found that on mathematical understanding and reasoning the students obtained better scores than that of the conventional group. The superiority of formal students than that of concrete students on mathematical understanding and reasoning taught by a metacognitive approach was reported by Nindiasari (2004).

The superiority of small group setting of learning (Think-Talk-Write, STAD, TAI, JIGSAW) from the conventional one is found in Ansyari (2004), Rahayu (2001), Sugandi (2001), and Wardani (2002) as well. It was found that students' abilities on mathematical communication, understanding and analogy, problem posing and solving were higher than those of conventional groups.

F. Findings of the Study and Discussion

For three years (2003-2005), the study had produced fourteen sets of learning materials based on the view of constructivism to investigate students' high level mathematical thinking abilities. Each experiment administered a set of non-routine essay mathematics test, a set of attitude scale, and an observation sheet of students' activities. Three sub-experiments by using indirect-direct approaches were carried out by Suryadi (2005) and Sumarni (2006) involving junior high school students, and Maya (2006) with senior high school students. Four other sub-experiments chose a problem-based learning with different subjects. Those include : (1) Ratnaningsih (2003) and Permana (2004) with senior high school students, and Herman (2006) with junior high school students. Seven sub-studies experimented different approaches with different groups of students. Mayadiana (2004) using a discursive approach worked with elementary school student teachers, and Darta (2004) using a contextual teaching approach worked with secondary high school student teacher. Muin (2005) experimented a metacognitive approach. Sukmadewi (2005) adopted a transactional reading strategy. Both of them worked with senior high school students. Mudzakir (2006) implemented Think-Talk-Write (TTW) strategy with junior high school students. The other two sub-studies use Information Communication Techology (ICT), including Yaniawati (2006) with e-learning for student teachers, and Kariadinata (2006) who used multimedia for senior high school students.

In general, all of the sub-studies found that the students' abillities of high level mathematical thinking taught with the new innovative approaches were classified as mediocre. However, experimental students obtained higher scores than those of conventional students (see Table 1). In relation to affective variables, the studies found that the new learning environment facilitated the growth of students' mathematical dispositions. During the lessons, students reinvented mathematical concepts actively, and had high interest in solving all mathematical tasks. Finally, it was found that teachers performed positive opinions toward the new approaches and they planed to implement them in their regular teaching.

Suryadi (2005) and Sumarni (2006) reported similar findings that indirect approach was more effective in improving students' higher level mathematical thinking than direct approach. A relatively different finding was reported by Maya (2006). Despite the fact that all students obtained good grade, she found that direct approach was more effective than indirect approach. It is probably due to the fact that Maya's students were classified as excellent students. However, Suryadi, Sumarni, and Maya reported similar findings that the combination of indirect-direct was the most effective among all.

Table 1 : High Level Mathematical Thinking Abilities of Students in Various Approaches and Strategies

Sub-experiments	Approaches strategies	Kinds of HLMTA*)	Number of Subjects	Students' high level mathematical thinking abilities					
				Pre-test/Prior knowledge			Post-test		
				Conventional Approach	Combinai of conventional and new appr	New Approaches/ strategies	Conventional Approach	Combinati of conventional and new appr	New Approaches/ strategies
Dewanto P. (2003)	Inductive-deductive	Math modeling	93 Under gradute Students	48 of 100	×	48 of 100	56 of 100	×	75 of 100
Darta (2003)	Contextual teaching	Problem solving	80 Student teachers	×	×	×	21,53 out of 44	×	30,51 out of 44
Mayadiana (2004)	Discursive approach**)	Critical thinking	36 Elementary stu. teachers	×	×	×	×	×	24 out of f 42 (mean from 4 cycluses)
Ratnaningsih (2003)	Problem based	Problem solving	Senior HS. students	76,3 out of 100	×	75,5 out of 100	7,2 out of 12	×	9,4 out of 12
Permana (2004)		Probl solving and connect	76 Senior HS students	70,71 out of 100	×	73,92 out of 100	25,50 out of 42	×	29,74 out of 42
Herman (2006)	Open and structured Problem based	All of HLMTA	90 Low, medium, and high junior HS	4,10 out of 20	4,38 out of 20	4,47 out of 20	7,15 out of 20	9,45 out of 20	9,32 out of 20
Mudzakir (2006)	Think-talk- write	Multiple representation	85 junior HS students	10,27 out of 80	×	10,71 out of 80	44,32 out of 80	×	55,12 out of 80
Muin (2005)	Metacognitive approach	All of HLMTA	65 Senior HS students	11,64 out of 65	×	14,28 out of 60	32,24 out of 60	×	39,53 out of 60
Suryadi (2005)	Indirect-direct approach	All of HLMTA	90 Low, medium, and high junior HS	42,17 out of 200	54,87 out of 200	47,93 out of 200	59,63 out of 200	94,27 out of 200	84,47 out of 200
Sumarni (2006)		All of HLMTA	125 Junior HS students	22 out of 100	35,75 out of 100	27,66 out of 100	43,37 out of 100	74,17 out of 100	52,22 out of 100
Maya (2006)		Comm, and reasoning	111 Senior HS students	22,2 out of 100	26,05 out of 100	32,15 out of 100	77,22 out of 100	81,50 out of 100	74,93 out of 100
Kariadinata (2006)	Multimedia	All of HLMTA	48 Senior HS students	×	×	×	56,60 out of 100	51,5 out of 100	55,12 out of 100
Yaniawati (2006)	e-learning	All of HLMTA	162 Students HS teacher	×	×	×	53,20 out of 100	56,75 out of 100	50,20 out of 100

Notes : *)HLMTA : high level mathematical thinking abilities
**)Four cycluses of classroom action research

Darta (2004), Dewanto (2003), Herman (2006), Mudzakir (2006), Muin (2006), Permana (2004), and Ratnaningsih (2003) found similar findings. They found that the mathematical abilities of students taught by using innovative approaches were better than those of conventional approaches. The other advantages of the innovative approaches included the followings : a) students of experimental groups explained their solutions reasonably, while the students of conventional groups worked more mechanically or procedurally (Dewanto, 2003), b) students of experimental groups had high self-directed learning and positive opinions to the new approaches (Darta, 2004, Herman, 2006, Mudzakir, 2006, Muin, 2006, Permana, 2004, and Ratnaningsih, 2003).

Suryadi and Herman further reported the similar findings that the innovative approaches were found to be more influential in improving students' mathematical abilities. This statement was supported by the findings that the abilities of students from low level school taught by using innovative approaches were better than those of students' from high level school taught by using conventional approach. However, considering gender, Suryadi and Herman reported different findings. Suryadi found that female students' abilities were better than those of male students, and Herman found there was no difference between female and male students' abilities.

Kariadinata (2006) and Yaniawati (2006) reported similar findings with others' that mathematical abilities of students taught by ICT were classified as mediocre. However, they reported different findings including : a) Mathematical abilities of experimental groups were lower than those of conventional groups, b) Students preferred combination of using ICT and conventional strategies, instead of only using ICT ; c) Students performed low self directed learning and they preferred to accept explanation instead of reinventing mathematical concepts or ideas ; d) Students had difficulties in operating the computer.

In relation to teaching process, almost all findings were in line with the theory that teaching and learning processes that encourage students' zone of proximal development (ZPD) would give opportunities to improve students' higher thinking abilities. Moreover, the studies pointed out that smarter students who were taught by the same approaches obtained better scores. These findings were in line with the theory that previous knowledge is a good predictor for successful learning. Nevertheless, the new approaches had some disadvantages : a) consuming more time, fund ; and teachers attempts, b) school limitation of ICT facilities, and c) teachers' difficulties in compiling learning materials especially for e-learning and multimedia approaches.

H. Conclusion

The conclusion of the study is stated as follows :

1) This study produced various models of learning materials and non-routine tests of high level mathematical thinking to fit with each teaching approach.

   The models of learning materials and tests could be modified to fit with students needs and curriculum objectives.

2) a. In general, students' abilities in mathematical thinking were classified as mediocre. However, students taught using innovative approaches obtained higher scores than those of the approaches. This statement is in line with the theory that teaching and learning process that encourage students' zone of proximal development (ZPD) would give opportunities of improving students' high level mathematical

thinking abilities.

b. Innovative learning conducted by researchers has been proven to be more important than other factors such as school categories and students' previous knowledge.

c. In relation to students' previous knowledge, the studies found that the smarter students performed higher mathematical abilities. However, inrelation to gender factor, the studies reported inconsistent findings.

d. Despite the fact that high level mathematical thinking tasks were difficult for students, they were important to be learned by students at any level of schools.

3) a. Students and teachers tended to have positive opinions toward the new innovative approaches. Students had high mathematical dispositions and teachers had willingness to implement the new approaches as well.

b. Other advantage of this study are that : researchers and school teachers were able to conduct collaborative studies to improve students' learning.

c. Some of the disadvantages are that : 1) The new approach consumed more time, facilities and extra teachers' attempts, 2) Teachers had some difficulties in compiling the learning materials of high level thinking and learning using multimedia.

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